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" A portion of the mound may still be seen overhanging the cliff, and if the section of it next the cliff be examined, the bones and charcoal may be easily observed.

" I gave the particulars of this discovery to Mr. D'Alton when he was about to publish his Memoir of Drogheda, and it is referred to in the first volume of his History of Drogheda. I stated to Mr. D'Alton that there was no tradition of the origin of this vast funereal pile, but he quotes a passage from Dr. Hanmer's Chronicles of Ireland, from which it would appear that a battle was fought between an army of marauders and Dermott Lamhdearg, King of Leinster, about the commencement of the fifth century, at Knock-na-cean, *i. e.*, the Hill of Heads, the marauders having landed at the 'Follesse of Skerries.'

" Rude stone coffins, composed of the common flag-stones of the country placed together in the form of a coffin, with skeletons, are found very frequently in this neighbourhood.

" Although it is unconnected with the foregoing, I may as well state, as a matter of curiosity, that Mr. Burdon, about the same time, when visiting the Hill of Tara, discovered and brought home to me a regular joint of a basaltic column, brought, no doubt, in the days of Tara's greatness from the Giant's Causeway. He discovered it accidentally; it was covered by the sod, and was not far from the pillar, supposed to be the Lia Fail."

Rev. Samuel Haughton, Fellow of Trinity College, read a paper on " The Equilibrium and Motion of elastic solid, and fluid Bodies."

The object of the paper is to deduce, by the method of the 'Mecanique Analytique' of Lagrange, the laws of solid and fluid bodies from the same physical principles, and to discover by the same method the conditions at the limits.

The principle from which Mr. Haughton deduces the equations is, that the molecules of solid and fluid bodies *act on each other in the direction of the line joining them*, with a force which is a function of the distance; and in the case of crystalline structure, also of the direction of the line joining the molecules.

The general equation of equilibrium of a system is,

$$\iiint(\mathrm{X}\delta\xi + \mathrm{Y}\delta\eta + \mathrm{Z}\delta\zeta)\, dm = \iiint \delta \mathrm{V} dxdydz. \qquad (1)$$

Mr. Haughton shows from the definition of the medium, that

$$\mathrm{V} = \mathrm{V}_0 + \mathrm{V}_1,$$

where V_0 is a homogeneous function of the first degree of the six quantities,

$$\frac{d\xi}{dx}, \frac{d\eta}{dy}, \frac{d\zeta}{dz}, \frac{d\eta}{dz} + \frac{d\zeta}{dy}, \frac{d\zeta}{dx} + \frac{d\xi}{dz}, \frac{d\xi}{dy} + \frac{d\eta}{dx}$$

and V_1 a homogeneous function of the second order of the same quantities. The function V_0 is zero in a solid body, and not in a fluid; which is equivalent to saying that in a solid body the molecular forces equilibrate each other without the aid of external forces, but that in fluids this is not the case, and that consequently a fluid left to itself would be dissipated by the action of its own molecular forces; this Mr. Haughton conceives to be true of all fluids, whether gaseous or liquid. It should be understood that the mutual *gravitations* of the particles are not included among the molecular forces, but among the external forces.

The values of V_0 and V_1 are shewn to be finally

$$\mathrm{V}_0 = p\left(\frac{d\xi}{dx} + \frac{d\eta}{dy} + \frac{d\zeta}{dz}\right) \qquad (2)$$

$$2\mathrm{V}_1 = \mathrm{A}\left(\frac{d\xi}{dx}\right)^2 + \mathrm{B}\left(\frac{d\eta}{dy}\right)^2 + \mathrm{C}\left(\frac{d\zeta}{dz}\right)^2 + \mathrm{L}u^2 + \mathrm{M}v^2 + \mathrm{N}w^2$$
$$+2\left(\mathrm{L}\frac{d\eta}{dy}\cdot\frac{d\zeta}{dz} + \mathrm{M}\frac{d\xi}{dx}\cdot\frac{d\zeta}{dz} + \mathrm{N}\frac{d\xi}{dx}\cdot\frac{d\eta}{dy}\right) + 2(\alpha_1 vw + \beta_2 uw + \gamma_3 uv)$$
$$+2\left\{u\left(\alpha_1\frac{d\xi}{dx} + \beta_1\frac{d\eta}{dy} + \gamma_1\frac{d\zeta}{dz}\right) + v\left(\alpha_2\frac{d\xi}{dx} + \beta_2\frac{d\eta}{dy} + \gamma_2\frac{d\zeta}{dz}\right)\right.$$
$$\left. + w\left(\alpha_3\frac{d\xi}{dx} + \beta_3\frac{d\eta}{dy} + \gamma_3\frac{d\zeta}{dz}\right)\right\}$$

where,

$$u = \frac{d\eta}{dz} + \frac{d\zeta}{dy}, \qquad v = \frac{d\zeta}{dx} + \frac{d\xi}{dz}, \qquad w = \frac{d\xi}{dy} + \frac{d\eta}{dx}.$$

It may be observed that the function V cannot be the same as the function used by Professor MacCullagh for light, which is a function of the quantities,

$$\frac{d\eta}{dz} - \frac{d\zeta}{dy}, \quad \frac{d\zeta}{dx} - \frac{d\xi}{dz}, \quad \frac{d\xi}{dy} - \frac{d\eta}{dx},$$

and that, consequently, if the latter represent light, that the molecules of the luminous ether cannot act on each other in the line joining them; in fact, the two functions mutually exclude each other.

From (2) is deduced

$$\iiint(\mathrm{X}\delta\xi + \mathrm{Y}\delta\eta + \mathrm{Z}\delta\zeta)\, dm = \tag{4}$$

$$\iint p\,\delta\xi\, dy\, dz + \iint p\,\delta\eta\, dx\, dz + \iint p\,\delta\zeta\, dx\, dy$$

$$- \iiint\left(\frac{dp}{dx}\,\delta\xi + \frac{dp}{dy}\,\delta\eta + \frac{dp}{dz}\,\delta\eta\right) dx\, dy\, dz,$$

which will give the well-known equations of hydrostatics, and the conditions at the limits of the fluid.

If the function V_1 be transformed by changing the directions of the axes of coordinates, it can be shown that the transformation is the same as the transformation of the surface

$$\mathrm{A}x^4 + \mathrm{B}y^4 + \mathrm{C}z^4 + 6(\mathrm{L}y^2z^2 + \mathrm{M}x^2z^2 + \mathrm{N}x^2y^2) \tag{5}$$

$$+ 4yz(3\alpha_1x^2 + \beta_1y^2 + \gamma_1z^2) + 4xz(\alpha_2x^2 + 3\beta_2y^2 + \gamma_2z^2)$$

$$+ 4xy(\alpha_3x^2 + \beta_3y^2 + 3\gamma_3z^2) = 1.$$

This surface may be called the characteristic surface of the function V_1, as, if it possesses any geometrical properties which simplify its equation, a corresponding simplification will take place in the function V_1.

It is then shown, by the aid of the auxiliary ellipsoid, whose equation is

$$(\mathrm{A}-\mathrm{L})x^2 + (\mathrm{B}-\mathrm{M})y^2 + (\mathrm{C}-\mathrm{N})z^2 + 2(\alpha_1 + \beta_1 + \gamma_1)yz$$
$$+ 2(\alpha_2 + \beta_2 + \gamma_2)xz + 2(\alpha_3 + \beta_3 + \gamma_3)xy = 1, \qquad (6)$$

that the function V_1 and the surface (5), may be referred to a system of rectangular axes, for which the following relations exist:

$$\aleph_1 + \beth_1 + \gimel_1 = 0; \quad \aleph_2 + \beth_2 + \gimel_2 = 0; \quad \aleph_3 + \beth_3 + \gimel_3 = 0; \qquad (7)$$

the Hebrew letters denoting what the Greek become after transformation of coordinates. The possibility of these equations in every case amounts to a proof of the existence of three axes at each point of a body, which are intimately connected with the molecular constitution of the body round the point.

The equation of equilibrium of a solid body is then shown to be

$$\int\!\!\int\!\!\int(\mathrm{X}\delta\xi + \mathrm{Y}\delta\eta + \mathrm{Z}\delta\zeta)dm = \triangle - \int\!\!\int\!\!\int(\mathrm{P}_1\delta\xi + \mathrm{Q}_1\delta\eta + \mathrm{R}_1\delta\zeta)dxdydz, \quad (8)$$

where

$$\mathrm{P}_1 = \mathrm{A}\frac{d^2\xi}{dx^2} + \mathrm{N}\frac{d^2\xi}{dy^2} + \mathrm{M}\frac{d^2\xi}{dz^2} + 2\left(\alpha_1\frac{d^2\xi}{dydz} + \alpha_2\frac{d^2\xi}{dxdz} + \alpha_3\frac{d^2\xi}{dxdy}\right)$$
$$+ \alpha_3\frac{d^2\eta}{dx^2} + \beta_3\frac{d^2\eta}{dy^2} + \gamma_3\frac{d^2\eta}{dz^2} + 2\left(\alpha_1\frac{d^2\eta}{dxdz} + \mathrm{N}\frac{d^2\eta}{dxdy} + \beta_2\frac{d^2\eta}{dydz}\right)$$
$$+ \alpha_2\frac{d^2\zeta}{dx^2} + \beta_2\frac{d^2\zeta}{dy^2} + \gamma_2\frac{d^2\zeta}{dz^2} + 2\left(\alpha_1\frac{d^2\zeta}{dxdy} + \gamma_3\frac{d^2\zeta}{dydz} + \mathrm{M}\frac{d^2\zeta}{dxdz}\right)$$

$$\mathrm{Q}_1 = \mathrm{B}\frac{d^2\eta}{dy^2} + \mathrm{L}\frac{d^2\eta}{dz^2} + \mathrm{N}\frac{d^2\eta}{dx^2} + 2\left(\beta_2\frac{d^2\eta}{dxdz} + \beta_3\frac{d^2\eta}{dxdy} + \beta_1\frac{d^2\eta}{dydz}\right)$$
$$+ \beta_1\frac{d^2\zeta}{dy^2} + \gamma_1\frac{d^2\zeta}{dz^2} + \alpha_1\frac{d^2\zeta}{dx^2} + 2\left(\beta_2\frac{d^2\zeta}{dxdy} + \mathrm{L}\frac{d^2\zeta}{dydz} + \gamma_3\frac{d^2\zeta}{dxdz}\right)$$
$$+ \beta_3\frac{d^2\xi}{dy^2} + \gamma_3\frac{d^2\xi}{dz^2} + \alpha_3\frac{d^2\xi}{dx^2} + 2\left(\beta_2\frac{d^2\xi}{dydz} + \alpha_1\frac{d^2\xi}{dxdz} + \mathrm{N}\frac{d^2\xi}{dxdy}\right)$$

$$\mathrm{R}_1 = \mathrm{C}\frac{d^2\zeta}{dz^2} + \mathrm{M}\frac{d^2\zeta}{dx^2} + \mathrm{L}\frac{d^2\zeta}{dy^2} + 2\left(\gamma_3\frac{d^2\zeta}{dxdy} + \gamma_1\frac{d^2\zeta}{dydx} + \gamma_2\frac{d^2\zeta}{dxdz}\right)$$
$$+ \gamma_2\frac{d^2\xi}{dz^2} + \alpha_2\frac{d^2\xi}{dx^2} + \beta_2\frac{d^2\xi}{dy^2} + 2\left(\gamma_3\frac{d^2\xi}{dydz} + \mathrm{M}\frac{d^2\xi}{dxdz} + \alpha_1\frac{d^2\xi}{dxdy}\right)$$
$$+ \gamma_1\frac{d^2\eta}{dz^2} + \alpha_1\frac{d^2\eta}{dx^2} + \beta_1\frac{d^2\eta}{dy^2} + 2\left(\gamma_3\frac{d^2\eta}{dxdz} + \beta_2\frac{d^2\eta}{dxdy} + \mathrm{L}\frac{d^2\eta}{dydz}\right),$$

Δ consists of double integrals, and gives the conditions at the limits.

The differential equations of *motion* derived from (8) are (no external forces X, Y, Z acting):

$$\varepsilon\frac{d^2\xi}{dt^2} = \mathrm{P}_1, \qquad \varepsilon\frac{d^2\eta}{dt^2} = \mathrm{Q}_1, \qquad \varepsilon\frac{d^2\zeta}{dt^2} = \mathrm{R}_1. \tag{9}$$

These equations will admit of the particular integral

$$\xi = \cos\alpha\,.f(\omega), \qquad \eta = \cos\beta\,.f(\omega), \qquad \zeta = \cos\gamma\,.f(\omega),$$

$$\omega = lx + my + nz - vt,$$

provided it is possible to satisfy with real values of $(\alpha, \beta, \gamma, v)$ the equations of condition resulting from the substitution of these values in the equations of motion.

These equations of condition lead to the following construction for the directions of the *possible* vibrations of molecules, and the corresponding velocities of wave-planes.

Construct the six *fixed* ellipsoids,

$$\left.\begin{aligned}
\mathrm{P} &= \mathrm{A}x^2 + \mathrm{N}y^2 + \mathrm{M}z^2 + 2\alpha_1 yz + 2\alpha_2 xz + 2\alpha_3 xy = 1,\\
\mathrm{Q} &= \mathrm{B}y^2 + \mathrm{L}z^2 + \mathrm{N}x^2 + 2\beta_1 yz + 2\beta_2 xz + 2\beta_3 xy = 1,\\
\mathrm{R} &= \mathrm{C}z^2 + \mathrm{M}x^2 + \mathrm{L}y^2 + 2\gamma_1 yz + 2\gamma_2 xz + 2\gamma_3 xy = 1,\\
\mathrm{F} &= \alpha_1 x^2 + \beta_1 y^2 + \gamma_1 z^2 + 2\mathrm{L}yz + 2\gamma_3 xz + 2\beta_2 xy = 1,\\
\mathrm{G} &= \alpha_2 x^2 + \beta_2 y^2 + \gamma_2 z^2 + 2\gamma_3 yz + 2\mathrm{M}xz + 2\alpha_1 xy = 1,\\
\mathrm{H} &= \alpha_3 x^2 + \beta_3 y^2 + \gamma_3 z^2 + 2\beta_2 yz + 2\alpha_1 xz + 2\mathrm{N}xy = 1,
\end{aligned}\right\} \tag{10}$$

and from their common centre draw the normal to the wave-plane, this will pierce the surfaces in six points; let the corresponding radii vectores be $\rho_{,}\,\rho_{,,}\,\rho_{,,,}$, $r_{,}\,r_{,,}\,r_{,,,}$; with these construct the ellipsoid

$$\frac{x^2}{\rho^2_{,}} + \frac{y^2}{\rho^2_{,,}} + \frac{z^2}{\rho^2_{,,,}} + \frac{2yz}{r^2_{,}} + \frac{2xz}{r^2_{,,}} + \frac{2xy}{r^2_{,,,}} = 1. \tag{11}$$

The axes of this ellipsoid will be the *three* possible directions of molecular vibration, and the corresponding velocities of waves will be inversely as the lengths of these axes.*

The six ellipsoids just mentioned perform a very important part in the problem of elastic solids, as they reappear in the conditions at the limits, and afford a geometrical meaning for many of the results.

Mr. Haughton then determines from simple considerations the equation of the Sphæro-Reciprocal-Polar of the Wave-surface, or the *Surface of Wave-slowness of elastic Solids*, which occupies a position in this subject, analogous to that held by the index-surface in light. This surface, and the important results it leads to, are, as far as Mr. Haughton is aware, given by him for the first time; it is of the sixth degree, and has three sheets, and by means of it, the direction of a vibration passing from one medium into another may be determined.

The paper then proceeds to the discussion of three particular cases of elastic solids: 1. The case where the molecules are arranged symmetrically round three rectangular planes. 2. Round one axis. 3. The case of a homogeneous uncrystalline body.

In the first case, the following results are deduced: The traces of the surface of wave-slowness on the planes of symmetry, consist of an ellipse and a curve of the fourth degree. The surface possesses four nodes in one of its principal planes, where the tangent plane becomes a cone of the second degree, and the existence of these points will give rise to a conical refraction in acoustics, similar to what has been established in physical optics.

In general, for a given direction of wave-plane, three waves will be possible, the corresponding vibrations of the molecules being in three directions, at right angles to each other, though not, in general, parallel or normal to the wave-plane. Mr. Haughton investigates the possibility of the vibrations being normal and transversal, and finds that for particular directions of wave (given in the paper), the vibrations are, two in the wave-plane, and the third perpendicular to it. He discusses also at length the other two cases, together with the equations of condition which hold in general at the limits, and the geometrical interpretation of these conditions by means of the fixed ellipsoids (10).

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* After Mr. Haughton had obtained this construction, he found that M. Cauchy has given analytically, and for a particular case, a solution which involves an analogous ellipsoid; but M. Cauchy has not followed out the consequences of his analysis in the right direction, and has been misled in his attempt to apply his equations to the problem of light.

The President read a note from Edward Cooper, Esq., giving the place of the new Comet, as observed at Markree, May 18d. 12h. 6m. Greenwich mean time.

DONATIONS.

Report of the Commissioners appointed to take the Census of Ireland for the Year 1841. Presented by W. R. Wilde, Esq.

Transactions of the Botanical Society of Edinburgh. Vol. II. Parts 1 & 2. Presented by the Society.

Lettres a S. A. R. le Duc Regnant de Saxe Coburg et Gotha, sur la Theorie des Probabilites appliquée aux Sciences morales et politiques. Par A. Quetelet. Presented by the Author.

Annales des Sciences Physiques et Naturelles, d'Agriculture et d'Industrie, publiées par la Société Royale d'Agriculture, &c., de Lyon. Tome VII. Annee 1844. Presented by the Society.

Popery unmasked. By John Ryan, Esq. Presented by the Author.

The thirteenth Annual Report of the Royal Cornwall Polytechnic Society. 1845. Presented by the Society.